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$$+(1n^{n+1}+2^{n+1}+3^{n+1}+4^{n+1}+\dots+n^{n+1})a_n.$$

Transposing the last quantity in parenthesis, reducing, and replacing the values of a_1 , a_2 , a_3 , we get,

$$\begin{aligned} & (1+2+3+\dots+n) + \frac{n}{2!}(1^2+2^2+3^2+\dots+n^2) + \frac{n(n-1)}{3!}(1^3+2^3+3^3+\dots+n^3) \\ & + \dots + \frac{n(n-1)}{3!}(1^{n-2}+2^{n-2}+3^{n-2}+\dots+n^{n-2}) + \frac{n}{2!}(1^{n-1}+2^{n-1}+3^{n-1} \\ & + \dots + n^{n-1}) + (1^n+2^n+3^n+\dots+n^n) = (n+1)^n - 1. \end{aligned}$$

When $n=2$, we get $(1+2)+(1^2+2^2)=3^2-1=8$.

When $n=3$, $(1+2+3)+\frac{3}{2}(1^2+2^2+3^2)+(1^3+2^3+3^3)=4^3-1=63$.

When $n=4$, $(1+2+3+4)+2(1^2+2^2+3^2+4^2)+2(1^3+2^3+3^3+4^3) \\ + (1^4+2^4+3^4+4^4)=5^4-1=624$.

When $n=5$, $(1+2+3+4+5)+\frac{5}{2}(1^2+2^2+3^2+4^2+5^2)+\frac{10}{3}(1^3+2^3+3^3 \\ +4^3+5^3)+\frac{5}{2}(1^4+2^4+3^4+4^4+5^4)+(1^5+2^5+3^5+4^5+5^5)=6^5-1=7775$.

When $n=6$, $(1+2+3+4+5+6)+3(1^2+2^2+3^2+4^2+5^2+6^2)+5(1^3+2^3+3^3 \\ +4^3+5^3+6^3)+5(1^4+2^4+3^4+4^4+5^4+6^4)+3(1^5+2^5+3^5+4^5+5^5+6^5)+(1^6+2^6+3^6 \\ +4^6+5^6+6^6)=7^6-1=117648$.

Also solved by *ELMER SCHUYLER*.

102. Proposed by *J. MARCUS BOORMAN*, Woodmere, N. Y.

$$\text{Solve } 2x + \sqrt{x^2 - 7} = 5.$$

I. Solution by *COOPER D. SCHMITT*, A. M., University of Tennessee, Knoxville, Tenn.; *M. A. GEUBER*, A. M., Washington, D. C.; *J. SCHEFFER*, A. M., Hagerstown, Md; and *G. B. M. ZERR*, A. M., Ph. D., Chester High School, Chester, Pa.

$$\text{Transposing, } \sqrt{x^2 - 7} = 5 - 2x.$$

$$\text{Squaring, } x^2 - 7 = 25 - 20x + 4x^2; \text{ whence } 3x^2 - 20x + 32 = 0.$$

$$\text{Solving, } x = 4 \text{ or } \frac{8}{3}.$$

Neither of these satisfies the original equation, but by writing it thus, $2x - \sqrt{x^2 - 7} = 5$, both values will satisfy it.

II. Solution by *H. C. WHITAKER*, M. E., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Denote $2x-5$ by y ; then the given equation reduces to

$$+ \sqrt{y^2 + 10y - 3} = -2y.$$

$$\text{Divide by } y, \quad + \sqrt{1 + 10/y - 3/y^2} = -2 \dots (1).$$

But this equation is absurd, since it makes a positive square root equal to a negative number.

Assume the symbolism $+ \sqrt{j} = -1$, j being an impossible quantity, the root of the equation $+ \sqrt{j}x + 1 = 0$.

Square (1), $1 + 10/y - 3/y^2 = 4j$, $y^2(4j - 1) - 10y + 3 = 0$, from which

$$x = \frac{10j \pm \sqrt{(7-3j)}}{4j-1}.$$

III. Solution by the PROPOSER.

Transpose and square.

$\therefore 4x^2 - 20x + 25 = x^2 - 7 \dots \dots (B)$, an obvious quadratic.

Apply its roots, 4 and $\frac{3}{4}$, to the given (A); hence $2(4) + [-3] = 8 - 3 = 5$;
 $\dots \dots = 2x + \{-[\sqrt{(16-7)}]\} \dots \dots (C)$; and

$$2(\frac{3}{4}) + (-\frac{3}{4}) = 5\frac{1}{4} - \frac{3}{4} = 5 \dots \dots = 2x + \{-[\sqrt{(\frac{64}{9} - \frac{9}{9})}]\} \dots \dots (D);$$

satisfy it. Could extracting $\sqrt{(x^2 - 7)}$ positive here also yield roots, then (A)'s dominant quadratic (B) is bi-quadratic, which is absurd.

Also solved by P. S. BERG and CHAS. C. CROSS.

GEOMETRY.

127. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the plane through the extremities, (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) , of conjugate diameters of the ellipsoid,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is } \frac{x_1 + x_2 + x_3}{a^2}x + \frac{y_1 + y_2 + y_3}{b^2}y + \frac{z_1 + z_2 + z_3}{c^2}z = 1.$$

Solution by the PROPOSER.

If $lx + my + nz = p \dots (1)$ be the required plane, we should have

$$lx_1 + my_1 + nz_1 = p \dots \dots (2),$$

$$lx_2 + my_2 + nz_2 = p \dots \dots (3),$$

$$lx_3 + my_3 + nz_3 = p \dots \dots (4).$$

Solving these for l/p , m/p , n/p , we have

$$l/p = \frac{\begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}} \div \dots \dots (5).$$

$$m/p = \text{etc.}, \quad n/p = \text{etc.}, \quad \dots \dots (6).$$

Reducing (5), making use of